Name: Ans Key

May 28 (A Block) May 30 (E Block), 2014

Algebra 2

Homework -- Probability: Expected Value

Definition: expected value

An *expected value* is an "average" value, calculated in the following way.
Supposed the possible values of a variable are *a*, *b*, *c*, etc., with probabilities $P(a)$, $P(b)$, $P(c)$, etc.

$$\text{Expected Value} = a \cdot P(a) + b \cdot P(b) + c \cdot P(c) + \cdots$$

In words: multiply each outcome by its probability, then take those products and add them.

Example of calculating an expected value:
Each night, your best friend phones you either 1, 2, or 3 times. There is a 35% of getting one call, a 40% chance of getting two calls, and a 25% chance of getting three calls. What is the expected number of phone calls?

$$\begin{aligned}\text{Expected value} &= 1 \cdot P(1\text{ call}) + 2 \cdot P(2\text{ calls}) + 3 \cdot P(3\text{ calls}) \\ &= 1 \cdot 0.35 + 2 \cdot 0.40 + 3 \cdot 0.25 = 1.9\end{aligned}$$

So, on average, you can expect to get 1.9 calls per night.

Problems:

1. Consider the number of loudspeaker announcements per day at school. Suppose there's a 15% chance of having 0 announcements, a 30% chance of having 1 announcement, a 25% chance of having 2 announcements, a 20% chance of having 3 announcements, and a 10% chance of having 4 announcements. Find the *expected value* of the number of announcements per day.

anouncements	0	1	2	3	4
Probability	0.15	0.30	0.25	0.20	0.10

$$\begin{aligned}EV &= 0(0.15) + 1(0.30) + 2(0.25) + 3(0.20) + 4(0.10) \\ &= 0 + .30 + 0.50 + 0.60 + 0.40 \\ &= 1.8\end{aligned}$$

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2. Suppose a coin is flipped 4 times, and let x equal the number of times that the coin is heads. The table shows the probabilities.

x = # of heads	probability
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

$$E.V. = 0\left(\tfrac{1}{16}\right) + 1\left(\tfrac{4}{16}\right) + 2\left(\tfrac{6}{16}\right) + 3\left(\tfrac{4}{16}\right) + 4\left(\tfrac{1}{16}\right)$$

$$= 0 + \frac{4}{16} + \frac{12}{16} + \frac{12}{16} + \frac{4}{16}$$

$$= \frac{32}{16} = 2$$

Calculate the *expected value* for the number of heads.

Games of chance

One common use for expected value is to understand the outcome of a game in which a player could either win or lose money. Winning outcomes are considered as positive values, and losing outcomes are considered as negative values. In this situation, here is how to interpret the expected value:

- If the expected value is positive, the game is favorable to the player.
- If the expected value is negative, the game is unfavorable to the player.
- If the expected value is zero, the game is called a "fair game."

3. Suppose there's a game where the player rolls a die, and either wins or loses money according to these rules.

die roll	1	2	3	4	5	6
outcome	lose \$5	lose \$5	lose \$5	win \$1	win \$1	win \$10

a. Let x = the amount of money that a player wins or loses in this game (where a loss is represented by a negative value of x). Complete this table of probabilities.

x = money won or lost	probability
–5	$\frac{3}{6} = \frac{1}{2}$
1	$\frac{2}{6} = \frac{1}{3}$
10	$\frac{1}{6}$

b. Calculate the expected value.

$$E.V. = -5\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{3}\right) + 10\left(\tfrac{1}{6}\right)$$

$$= -0.50$$

c. For the player, is this game favorable, unfavorable, or a "fair game"?

unfavorable

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4. A state runs a lottery called "Pick 6." A player buys a lottery ticket showing 6 of the numbers from 1 through 40. Then a set of 6 numbers is chosen as the winning numbers, and anyone whose ticket matches those numbers wins a prize. (The order of the numbers does not matter.)

a. What is the probability that you pick the winning numbers?

\# of Combinations is ${}_{40}C_6$ =

b. What is the probability that you do NOT pick the winning numbers?

Suppose that a lottery ticket costs \$1, and the prize for having a winning ticket is \$1,000,000.

c. Calculate the expected value for a lottery play.

d. For the player, is this lottery favorable, unfavorable, or a "fair game"?

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5. A "litter" is a set of puppies born from the same mother at the same time. Suppose that a dog breeder kept track of the sizes of each litter, and recorded this data:

number of puppies	how many litters	Probability
1	1	1/40
2	3	3/40
3	3	3/40
4	4	4/40
5	5	5/40
6	7	7/40
7	9	9/40
8	5	5/40
9	0	0/40
10	2	2/40
11	1	1/40

Answer these questions based on the above data. $\Sigma = 40$

a. What number of puppies occurred most often? What is the probability that a litter contained that number of puppies?

the number of puppies that occurred most often was 7.
the probability that a litter contained 7 puppies is $\frac{9}{40} = 0.225$

b. What is the probability of having 10 or more puppies in a litter?

$$P(\geq 10) = \frac{2+1}{40} = \frac{3}{40} = 0.075$$

c. What is the probability of having fewer than 4 puppies in a litter?

$$P(<4) = \frac{7}{40} = 0.175$$

d. What is the average number of puppies in a litter? In other words, find the *expected value* of the number of puppies.

$$EV = 1\left(\tfrac{1}{40}\right) + 2\left(\tfrac{3}{40}\right) + 3\left(\tfrac{3}{40}\right) + 4\left(\tfrac{4}{40}\right) + 5\left(\tfrac{5}{40}\right) + 6\left(\tfrac{7}{40}\right) + 7\left(\tfrac{9}{40}\right) + 8\left(\tfrac{5}{40}\right) + 9\left(\tfrac{0}{40}\right) + 10\left(\tfrac{2}{40}\right) + 11\left(\tfrac{1}{40}\right)$$

$$= \frac{1}{40} + \frac{6}{40} + \frac{9}{40} + \frac{16}{40} + \frac{25}{40} + \frac{42}{40} + \frac{63}{40} + \frac{40}{40} + \frac{0}{40} + \frac{20}{40} + \frac{11}{40}$$

$$= \frac{233}{40} = 5.825$$