Accelerated Math 3
More Trig Form of a Complex Number

Name Key

Perform the operation and leave the result in trigonometric form.

1. $\left[3\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]$ $12\left(\cos\pi/2+i\sin\pi/2\right)$

2. $\left[\frac{3}{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]\left[6\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]$ $9\left(\cos 5\pi/12+i\sin 5\pi/12\right)$

3. $\left[\frac{5}{3}\left(\cos 140^\circ+i\sin 140^\circ\right)\right]\left[\frac{2}{3}\left(\cos 60^\circ+i\sin 60^\circ\right)\right]$ $\frac{10}{9}\left(\cos 200^\circ+i\sin 200^\circ\right)$

4. $\left[\frac{1}{2}\left(\cos 115^\circ+i\sin 115^\circ\right)\right]\left[\frac{4}{5}\left(\cos 300^\circ+i\sin 300^\circ\right)\right]$ $\frac{2}{5}\left(\cos 55^\circ+i\sin 55^\circ\right)$

5. $\dfrac{\cos 50^\circ+i\sin 50^\circ}{\cos 20^\circ+i\sin 20^\circ}$ $\cos 30^\circ+i\sin 30^\circ$

6. $\dfrac{5(\cos 4.3+i\sin 4.3)}{4(\cos 2.1+i\sin 2.1)}$ $\frac{5}{4}\left(\cos 2.2+i\sin 2.2\right)$

Use DeMoivre's Theorem to find the indicated power of the complex number. Express the result in standard form.

7. $(1+i)^3$ $r=\sqrt{2}$, $\tan\theta=1$, $\theta=\pi/4$ $2\sqrt{2}\left(\cos 3\pi/4+i\sin 3\pi/4\right)$ $(-\sqrt{2}/2+\sqrt{2}/2\,i)$ $-2+2i$

8. $(2+2i)^6$ $r=\sqrt{8}$, $\tan\theta=1$, $\theta=\pi/4$ $512\left(\cos 6\pi/4+i\sin 6\pi/4\right)$ $-512i$

9. $(-1+i)^{10}$ $r=\sqrt{2}$, $\tan\theta=-1$, $\theta=3\pi/4$ $32\left(\cos 30\pi/4+i\sin 30\pi/4\right)$ $\left(\cos 3\pi/2+i\sin 3\pi/2\right)$ $-32i$

10. $(1-i)^{12}$ $r=\sqrt{2}$, $\tan\theta=-1$, $\theta=7\pi/4$ $64\left(\cos 21\pi+i\sin 21\pi\right)$ -64

11. $2(\sqrt{3}+i)^5$ $r=\sqrt{4}$, $r=2$, $\tan\theta=1/\sqrt{3}$, $\theta=\pi/6$ $2^5\left(\cos 5\pi/6+i\sin 5\pi/6\right)$ $32\left(-\sqrt{3}/2+\frac{1}{2}i\right)$ $-32\sqrt{3}+32i$

12. $4(1-\sqrt{3}i)^3$ $r=2$, $\tan\theta=-\sqrt{3}$, $\theta=5\pi/3$ $8\left(\cos 5\pi+i\sin 5\pi\right)$ $-1\quad\ 0$ -32

13. $[5(\cos 20^\circ+i\sin 20^\circ)]^3$ $125\left(1/2-\sqrt{3}/2\,i\right)$ $\frac{125}{2}-\frac{125\sqrt{3}}{2}i$

14. $[3(\cos 150^\circ+i\sin 150^\circ)]^4$ $81\left(\cos 600^\circ+i\sin 600^\circ\right)$ $-\frac{1}{2}\quad +-\frac{\sqrt{3}}{2}$ $-\frac{81}{2}-\frac{81\sqrt{3}}{2}i$

15. $[4(\cos 2.8+i\sin 2.8)]^5$ $1024\left(-.137+.991i\right)$ $-140.288+1014.784i$

16. $(\cos 0+i\sin 0)^{20}$ 1

Find the indicated roots of the complex number. Express each of the roots in standard form.

17. Square roots of $5(\cos 120^\circ + i \sin 120^\circ)$

$\sqrt{5}\left(\cos \frac{120^\circ + 360^\circ k}{2} + i \sin \frac{120^\circ + 360^\circ k}{2}\right)$

$k=0$ $\sqrt{5}(\cos 60^\circ + i \sin 60^\circ)$ $\quad \frac{\sqrt{5}}{2} + \frac{\sqrt{15}}{2} i$

$k=1$ $\sqrt{5}(\cos 240^\circ + i \sin 240^\circ)$ $\quad -\frac{\sqrt{5}}{2} - \frac{\sqrt{15}}{2} i$

18. Fourth roots of $16\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

$2\left(\cos \frac{4\pi/3 + 2\pi k}{4} + i \sin \frac{4\pi/3 + 2\pi k}{4}\right)$

$k=0$ $2(\text{cis } \pi/3)$ $\quad 1 + \sqrt{3} i$

$k=1$ $2(\text{cis } 5\pi/6)$ $\quad -\sqrt{3} + i$

$k=2$ $2(\text{cis } 4\pi/3)$ $\quad -1 - \sqrt{3} i$

$k=3$ $2(\text{cis } 11\pi/6)$ $\quad \sqrt{3} - i$

19. Cube roots of $-27i$ $\quad r=27$, $\theta = 3\pi/2$

$3\left(\cos \frac{3\pi/2 + 2\pi k}{3} + i \sin \frac{3\pi/2 + 2\pi k}{3}\right)$

$k=0$ $3(\text{cis } \pi/2)$ $\quad 3i$

$k=1$ $3(\text{cis } 7\pi/6)$ $\quad -\frac{3\sqrt{3}}{2} - \frac{3}{2} i$

$k=2$ $3(\text{cis } 11\pi/6)$ $\quad \frac{3\sqrt{3}}{2} - \frac{3}{2} i$

20. Cube roots of $-4\sqrt{2}(1 - i)$ $\quad r=8$, $\theta = 7\pi/4$

21. Fourth roots of i

22. Fifth roots of i

23. Cube roots of 1000

24. Fourth roots of - 4

Find all solutions of the equation.

25. $x^4 - i = 0$ $\quad r=1$, $\theta = \pi/2$

$x^4 = 0 + i$

$k=0$ $1(\text{cis } \pi/8)$

$k=1$ $1(\text{cis } 5\pi/8)$

$k=2$ $1(\text{cis } 9\pi/8)$

$k=3$ $1(\text{cis } 13\pi/8)$ $\quad .924 + .382i$, $-.382 + .924i$, $-.924 - .382i$, $.382 - .924i$

26. $x^3 + 27 = 0$ $\quad r=27$, $\theta = \pi$

$x^3 = -27$

$k=0$ $3(\text{cis } \pi/3)$ $\quad 3/2 + 3\sqrt{3}/2\, i$

$k=1$ $3(\text{cis } \pi)$ $\quad -3$

$k=2$ $3(\text{cis } 5\pi/3)$ $\quad 3/2 - 3\sqrt{3}/2\, i$

27. $x^5 - 243 = 0$

28. $x^4 + 81 = 0$

29. $x^3 + 64i = 0$

30. $x^6 - 64i = 0$