Radical & Rational Function Review

1. $\frac{x-6y}{12y^2x^3} - \frac{x-4y}{12y^2x^3} = \frac{x-6y-x+4y}{12y^2x^3}$

$= \frac{-2y}{12y^2x^3} = \boxed{\frac{-1}{6yx^3}}$ $\quad \boxed{\begin{matrix} x\neq 0 \\ y\neq 0 \end{matrix}}$

2. $\frac{m+2n}{20n^2} + \frac{m-5n}{20n^2} = \frac{m+2n+m-5n}{20n^2} = \boxed{\frac{2m-3n}{20n^2}}$

$\boxed{n\neq 0}$

3. $\frac{8}{2n} + \frac{3n-7}{4n^2+28n} = \frac{8}{2n} + \frac{3n-7}{4n(n+7)}$ $\quad LCD = 4n(n+7)$

$\frac{8(2)(n+7)}{2n(2)(n+7)} + \frac{3n-7}{4n(n+7)} = \frac{16(n+7)+3n-7}{4n(n+7)} = \frac{16n+112+3n-7}{4n(n+7)}$

$= \boxed{\frac{19n+105}{4n(n+7)}}$ $\quad \boxed{\begin{matrix} n\neq 0 \\ n\neq -7 \end{matrix}}$

4. $\frac{2}{x-5} + \frac{3}{x+5}$ $\quad LCD = (x-5)(x+5)$ $\quad \frac{5(x-1)}{(x+5)(x-5)}$

$= \frac{2(x+5)}{(x+5)(x-5)} + \frac{3(x-5)}{(x+5)(x-5)} = \frac{2x+10+3x-15}{(x+5)(x-5)} = \boxed{\frac{5x-5}{(x+5)(x-5)}}$

$\boxed{x\neq \pm 5}$

5. $\frac{6}{2k+2} - \frac{8k}{k+2} = \frac{6}{2(k+1)} - \frac{8k}{(k+2)}$ LCD $= 2(k+1)(k+2)$

$\frac{6(k+2)}{2(k+1)(k+2)} - \frac{8k(2)(k+1)}{2(k+1)(k+2)} = \frac{6k+12}{2(k+1)(k+2)} - \frac{16k(k+1)}{2(k+1)(k+2)}$

$= \frac{6k+12}{2(k+1)(k+2)} - \frac{16k^2+16k}{2(k+1)(k+2)} = \frac{6k+12-16k^2-16k}{2(k+1)(k+2)}$

$= \frac{-16k^2-10k+12}{2(k+1)(k+2)} = -\frac{(16k^2+10k-12)}{2(k+1)(k+2)} = -\frac{\cancel{2}(8k^2+5k-6)}{\cancel{2}(k+1)(k+2)}$

$= \boxed{\frac{-(8k^2+5k-6)}{(k+1)(k+2)}}$ or $\boxed{\frac{-8k^2-5k+6}{(k+1)(k+2)}}$ $\boxed{\begin{array}{l} k \neq -1 \\ k \neq -2 \end{array}}$

6. $\frac{6a}{2a} - \frac{4}{3a^2+39a+126} = \frac{6a}{2a} - \frac{4}{3(a^2+13a+42)}$

$= \frac{6}{2} - \frac{4}{3(a+6)(a+7)}$ LCD $= 2(3)(a+6)(a+7)$

$\frac{6(3)(a+6)(a+7)}{2(3)(a+6)(a+7)} - \frac{4(2)}{2(3)(a+6)(a+7)} = \frac{18(a+6)(a+7)-8}{2(3)(a+6)(a+7)}$

$= \frac{18(a^2+13a+42)-8}{(2)(3)(a+6)(a+7)} = \frac{18a^2+234a+756-8}{(2)(3)(a+6)(a+7)}$

$= \frac{18a^2+234a+748}{2(3)(a+6)(a+7)} = \frac{\cancel{2}(9a^2+117a+374)}{\cancel{2}(3)(a+6)(a+7)} =$

$\boxed{\frac{9a^2+117a+374}{3(a+6)(a+7)}}$ $\boxed{\begin{array}{l} a \neq -6 \\ a \neq -7 \\ a \neq 0 \end{array}}$

7. $\dfrac{20}{\frac{2}{5}-\frac{25}{4}} = \dfrac{20}{\frac{8}{20}-\frac{125}{20}} = \dfrac{20}{\frac{-117}{20}} = \dfrac{\frac{20}{1}}{\frac{-117}{20}} = \dfrac{20}{1}\cdot\dfrac{20}{-117}$

$= \boxed{\dfrac{400}{-117}}$

8. $\dfrac{\frac{15}{4}+\frac{1}{3}}{3} = \dfrac{\frac{45}{12}+\frac{4}{12}}{3} = \dfrac{\frac{49}{12}}{\frac{3}{1}} = \dfrac{49}{12}\cdot\dfrac{1}{3} = \boxed{\dfrac{49}{36}}$

9. $\dfrac{\frac{5}{k^2}-\frac{k}{4}}{25} = \dfrac{\frac{20}{4k^2}-\frac{k^3}{4k^2}}{25} = \dfrac{\frac{20-k^3}{4k^2}}{\frac{25}{1}} = \dfrac{20-k^3}{4k^2}\cdot\dfrac{1}{25}$

$= \boxed{\dfrac{20-k^3}{100k^2}} \quad \boxed{k\neq 0}$

10. $\dfrac{4}{\frac{u-1}{3}+\frac{16}{u-1}} = \dfrac{4}{\frac{(u-1)(u-1)}{3(u-1)}+\frac{16(3)}{3(u-1)}} = \dfrac{4}{\frac{u^2-2u+1+48}{3(u-1)}} = \dfrac{4}{\frac{u^2-2u+49}{3(u-1)}}$

$= \dfrac{4}{1}\cdot\dfrac{3(u-1)}{u^2-2u+49} = \boxed{\dfrac{12(u-1)}{u^2-2u+49}} \quad \boxed{u\neq 1}$

* The other roots are complex

11. $\dfrac{\frac{5}{4}-\frac{25}{4}}{\frac{4}{k^2}-\frac{4}{25}} = \dfrac{\frac{-20}{4}}{\frac{100}{25k^2}-\frac{4k^2}{25k^2}} = \dfrac{-5}{\frac{100-4k^2}{25k^2}} = \dfrac{-5}{1}\cdot\dfrac{25k^2}{100-4k^2}$

$= \boxed{\dfrac{-125k^2}{100-4k^2}} \quad \boxed{k\neq 0}$

12) $$\frac{\frac{a}{3}+\frac{a}{25}}{\frac{1}{3}-\frac{a-4}{9}} = \frac{\frac{25a}{75}+\frac{3a}{75}}{\frac{3}{9}-\frac{a-4}{9}} = \frac{\frac{28a}{75}}{\frac{3-a+4}{9}} = \frac{\frac{28a}{75}}{\frac{7-a}{9}}$$

$$= \frac{28a}{75}\cdot\frac{9}{7-a} = \frac{252a}{525-75a} = \frac{3(84a)}{3(175-25a)} = \boxed{\frac{84a}{175-25a}} \quad \boxed{a\neq 7}$$

13) $$\frac{4}{17}\cdot\frac{19n}{9} = \boxed{\frac{76n}{153}}$$

14) $$\frac{10}{19b}\cdot\frac{2b^2}{5} = \frac{20b^2}{95b} = \boxed{\frac{4b}{19}} \quad \boxed{b\neq 0}$$

15) $$\frac{x^2+7x+6}{7}\cdot\frac{1}{x+6} = \frac{(x+6)(x+1)}{7(x+6)} = \boxed{\frac{x+1}{7};\ x\neq -6}$$

16) $$\frac{v+5}{v^2+13v+40}\cdot\frac{6v+48}{7} = \frac{(v+5)(6)(v+8)}{(v+8)(v+5)(7)} = \boxed{\frac{6}{7},\ x\neq -8,\ x\neq -5}$$

17) $$\frac{18a^2+72a}{10a^2+20a}\cdot\frac{2a^2+14a+20}{18a^2+72a} = \frac{18a(a+4)}{10a(a+2)}\cdot\frac{(2)(a^2+7a+10)}{18a(a+4)}$$

$$= \frac{2(a+5)(a+2)}{10a(a+2)} = \frac{(a+5)}{5\cdot 10a} = \boxed{\frac{a+5}{5a};\ x\neq -2,-4,0}$$

18) $$\frac{3n+10}{12}\cdot\frac{(2n+1)(n-5)}{(2n+1)(3n+10)} = \boxed{\frac{n-5}{12};\ n\neq\frac{1}{2},\ n\neq -\frac{10}{3}}$$

19) LCD $= 3x^2$

$$3x^2\left(\frac{x-4}{3x^2} = \frac{1}{3x}+\frac{1}{x}\right) = \frac{3x^2(x-4)}{3x^2} = \frac{3x}{3x}+\frac{3x^2}{x}$$

$$= x-4 = x+3x \Rightarrow x-4=4x \Rightarrow -4=3x \Rightarrow \boxed{x=-\frac{4}{3}}$$

CKV

20) $\frac{1}{6v^2} - \frac{5}{6v} = \frac{2}{3v^2}$ LCD $= 6v^2$

$6v^2\left(\frac{1}{6v^2} - \frac{5}{6v} = \frac{2}{3v^2}\right) = \frac{6v^2}{6v^2} - \frac{30v^2}{6v} = \frac{12v^2}{3v^2} \Rightarrow 1 - 5v = 4$

$-5v = 3 \Rightarrow \boxed{v = -\frac{3}{5}}$ ck ✓

$\boxed{v \neq 0}$

21) $\frac{1}{n+6} + \frac{6}{n(n+6)} = \frac{1}{n(n+6)}$ LCD $= n(n+6)$

$n(n+6)\left(\frac{1}{n+6} + \frac{6}{n(n+6)} = \frac{1}{n(n+6)}\right) = \frac{n(n+6)}{(n+6)} + \frac{6n(n+6)}{n(n+6)} = \frac{n(n+6)}{n(n+6)}$

$n + 6 = 1 \Rightarrow n = 1 - 6 \Rightarrow \boxed{n = -5}$ ck ✓ $\boxed{\begin{array}{l} n \neq 0 \\ n \neq -6 \end{array}}$

22) $\frac{6}{k+2} = \frac{1}{k+2} - 1$ LCD $= k+2$

$\frac{(k+2)6}{k+2} = \frac{k+2(1)}{k+2} - \frac{(k+2)(1)}{1}$

$6 = 1 - (k+2) \Rightarrow 6 = 1 - k - 2 \Rightarrow 6 = -k - 1 \Rightarrow$

$7 = -k \Rightarrow \boxed{-7 = k}$ ck ✓ $\boxed{k \neq -2}$

23) $\frac{3k}{2(k+2)} + \frac{1}{4k(k+2)} = \frac{1}{4k}$ LCD $= 4k(k+2)$

$\frac{4k(k+2)(3k)}{2(k+2)} + \frac{4k(k+2)}{4k(k+2)} = \frac{4k(k+2)}{4k}$

$\boxed{\begin{array}{l} k \neq 0 \\ k \neq -2 \end{array}}$

$= 2k(3k) + 1 = k + 2 \Rightarrow 6k^2 + 1 = k + 2 \Rightarrow 6k^2 - k - 1 = 0$

$\Rightarrow (3k+1)(2k-1) = 0$ $\boxed{k = -\frac{1}{3},\ k = \frac{1}{2}}$ ck ✓

24) $\frac{1}{2} = \frac{5}{2p(p-4)} + \frac{(p+5)(p-5)}{p(p-4)}$ LCD: $2p(p-4)$

$\frac{\cancel{2}p(p-4)}{\cancel{2}} = \frac{5(2)p(p-4)}{2p(p-4)} + \frac{2p(p-4)(p+5)(p-5)}{p(p-4)}$

$= p^2 - 4p = 5 + 2(p+5)(p-5)$

$= p^2 - 4p = 5 + 2(p^2 - 25) \Rightarrow p^2 - 4p = 5 + 2p^2 - 50$

$0 = p^2 + 4p - 45$

$\Rightarrow 0 = (p+9)(p-5) \Rightarrow$ **$p = -9 \quad p = 5$** ✓ck

$p \neq 0$, $p \neq 4$

25) $\frac{k+5}{2k+10} = \frac{k+5}{2(k+5)} =$ **$\frac{1}{2}$; $k \neq -5$**

26) $\frac{8n^3}{6n^2 - 10n} = \frac{4 \cdot 2 n^2 \cdot n}{2n(3n-5)} =$ **$\frac{4n^2}{3n-5}$; $n \neq \frac{5}{3}$, $n \neq 0$**

27) $\frac{r^2 + 14r + 48}{r^2 + 5r - 6} = \frac{(r+8)(r+6)}{(r+6)(r-1)} =$ **$\frac{r+8}{r-1}$; $r \neq -6$, $r \neq 1$**

28) $\frac{8m^3 - 32m^2}{m^2 - 5m + 4} = \frac{8m^2(m-4)}{(m-4)(m-1)} =$ **$\frac{8m^2}{m-1}$; $m \neq 4$, $m \neq 1$**

29) $\frac{11}{13k} \cdot \frac{6k^3}{19} = \frac{66k^3}{247k} =$ **$\frac{66k^2}{247}$ $k \neq 0$**

30) $\frac{7}{5n^2} \cdot \frac{14}{10} = \frac{98}{50n^2} =$ **$\frac{49}{25n^2}$; $n \neq 0$**

31) $\frac{7v^2+14v}{2v} \div \frac{7v^2+7v}{v+1} = \frac{7v(v+2)}{2v} \cdot \frac{(v+1)}{7v(v+1)}$

$= \boxed{\frac{v+2}{2v} \; ; \; v \neq -1, 0}$

32) $\frac{4}{8b^2+20b} \div \frac{4}{16b+40} = \frac{4}{4b(2b+5)} \cdot \frac{8(2b+5)}{4}$

$= \boxed{\frac{2}{b} \; ; \; b \neq 0, \; b \neq -\frac{5}{2}}$

33) $\frac{5k^2+15k-50}{12k+40} \div \frac{10k-20}{3k+10} = \frac{5(k^2+3k-10)}{4(3k+10)} \cdot \frac{3k+10}{10(k-2)}$

$= \frac{5(k+5)(k-2)}{4(3k+10)} \cdot \frac{3k+10}{10(k-2)} = \boxed{\frac{k+5}{8} \; ; \; k \neq -\frac{10}{3}, \; k \neq 2}$

34) $\frac{2n^2-23n+56}{8n^2} \cdot \frac{8n^3+40n^2}{16n^2-56n}$

$= \frac{(2n-7)(n-8)}{8n^2} \cdot \frac{8n^2(n+5)}{8n(2n-7)}$

$= \boxed{\frac{(n-8)(n+5)}{8n} \; ; \; n \neq 0, \; n \neq 7/2}$

35) $\frac{20a+4}{8a} = \frac{4(5a+1)}{4(2a)} = \boxed{\frac{5a+1}{2a}\ ;\ a \neq 0}$

36) $\frac{16k}{8k^2+12k} = \frac{4(4k)}{4k(2k+3)} = \boxed{\frac{4}{2k+3}\ ;\ k \neq 0,\ k \neq -\frac{3}{2}}$

37) $\frac{k^2+5k-24}{k^2+16k+64} = \frac{(k-3)(k+8)}{(k+8)(k+8)} = \boxed{\frac{k-3}{k+8}\ ;\ k \neq -8}$

38) $\frac{6p-10}{14p+10} = \frac{2(3p-5)}{2(7p+5)} = \boxed{\frac{3p-5}{7p+5}\ ;\ p \neq \frac{-5}{7}}$

39) $\frac{42n^2+18n}{42n^3+42n^2-84n} = \frac{6n(7n+3)}{6n(7n^2+7n-14)}$

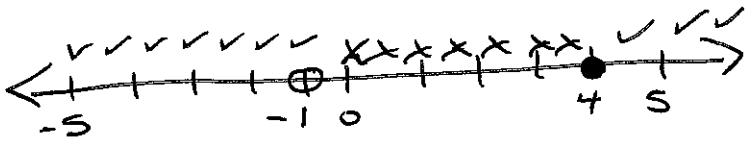
$= \frac{7n+3}{7(n^2+n-2)} = \boxed{\frac{7n+3}{7(n+2)(n-1)}\ ;\ k \neq 0, -2, 1}$

40) $\frac{28m^2+72m+32}{28m-40} = \frac{4(7m^2+18m+8)}{4(7m-10)}$

$= \frac{4(7m+4)(m+2)}{4(7m-10)} = \boxed{\frac{(7m+4)(m+2)}{(7m-10)}\ ;\ m \neq \frac{10}{7}}$

41) $\frac{x-4}{x+1} \geq 0$

x	y	$y/N \geq 0$
-2	6	✓
0	-4	✗
6	$\frac{2}{7} \approx .2857$	✓



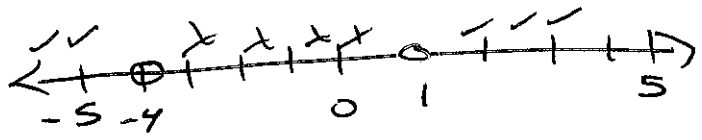
$\boxed{(-\infty, -1) \cup [4, \infty)}$

42) $\frac{-x+6}{x+4} < 1 \Rightarrow \frac{-x+6}{x+4} - \frac{1}{1} < 0$

$\Rightarrow \frac{-x+6}{x+4} - \frac{x+4}{x+4} < 0 \Rightarrow \frac{-x+6-x-4}{x+4} < 0$

$\Rightarrow \frac{-2x+2}{x+4} < 0$

x	y	$TIF < 0$
-5	-12	T ✓
0	$\frac{1}{2}$	F ✗
2	$-\frac{1}{3}$	T ✓



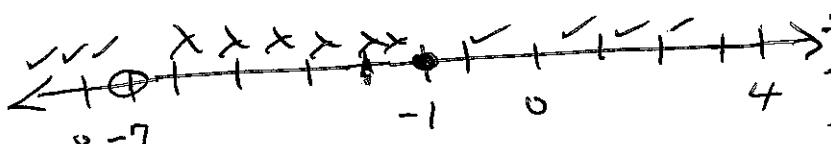
$\boxed{(-\infty, -4) \cup (1, \infty)}$

43) $\frac{5x-49}{3x+21} \geq -3 \Rightarrow \frac{5x-49}{3x+21} + \frac{3}{1} \geq 0$

$\Rightarrow \frac{5x-49}{3x+21} + \frac{3(3x+21)}{3x+21} \geq 0 \Rightarrow \frac{5x-49+9x+63}{3x+21} \geq 0$

$\Rightarrow \frac{14x+14}{3x+21} \geq 0$

x	y	$TIF \geq 0$
-8	32.7	T ✓
-2	$-.93$	F ✗
0	$\frac{2}{3}$	T ✓

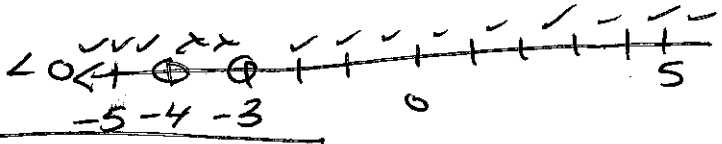


$\boxed{(-\infty, -7) \cup [1, \infty)}$

44. $\frac{x+7}{x+4} < 4 \Rightarrow \frac{x+7}{x+4} - \frac{4}{1} < 0$

$\Rightarrow \frac{x+7}{x+4} - \frac{4(x+4)}{(x+4)} < 0 \Rightarrow \frac{x+7-4x-16}{x+4}$

$\Rightarrow \frac{-3x-9}{x+4} < 0$



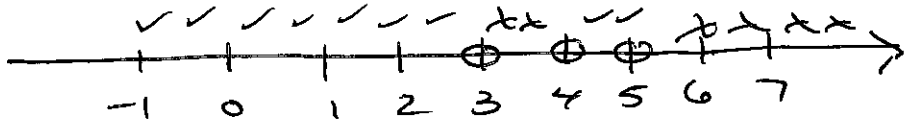
x	y	TF <0
-5	-6	✓
-3.5	8	x
0	-23	✓

$(-\infty, -4) \cup (-3, \infty)$

45. $\frac{2}{x-5} < \frac{1}{x-4} \Rightarrow \frac{2}{x-5} - \frac{1}{x-4} < 0$

$\Rightarrow \frac{2(x-4)}{(x-4)(x-5)} - \frac{1(x-5)}{(x-4)(x-5)} < 0 \Rightarrow \frac{2x-8-x+5}{(x-4)(x-5)} < 0$

$\frac{x-3}{(x-4)(x-5)} < 0$



x	y	TF <0
0	-1.5	T ✓✓
3.5	.66	F x x x
4.5	-6	T ✓✓
6	1.5	F x

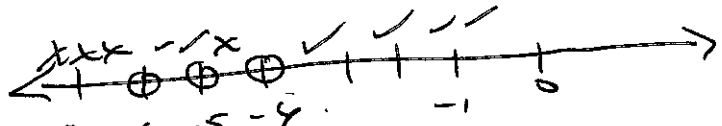
$(-\infty, +3) \cup (4, 5)$

46) $-\frac{1}{x+5} > -\frac{2}{x+6} \Rightarrow \frac{-1}{x+5} + \frac{2}{x+6} > 0$

$\frac{-1(x+6)+2(x+5)}{(x+5)(x+6)} > 0 \Rightarrow \frac{-x-6+2x+10}{(x+5)(x+6)} > 0$

x	y	TF >0
-7	-1.5	F x
-5.5	6	T ✓
-4.5	-.66	F x
0	.133	✓ T

$\frac{x+4}{(x+5)(x+4)} > 0$



$(-6, -5) \cup (-4, \infty)$

47)

a) $I = M + B$

$$= \frac{28,390 + 693t}{85 - t} + \frac{776 - 12t}{55 - 2t}$$

$$= \frac{(28,390 + 693t)(55 - 2t)}{(85 - t)(55 - 2t)} + \frac{(776 - 12t)(85 - t)}{(85 - t)(55 - 2t)}$$

$$= \frac{1,561,450 - 56,780t + 38,115t - 1386t^2 + 65,960 - 776t - 1,020t + 12t^2}{(85 - t)(55 - 2t)}$$

$$= \boxed{\frac{-1,374t^2 - 20,461t + 1,627,410}{(85 - t)(55 - 2t)}}$$

b) $t = 15$

$$= \frac{-1374(15)^2 - 20,461(15) + 1627410}{(85 - 15)(55 - 2(15))}$$

$$= \frac{1011345}{1750}$$

$$= 578 \text{ doctors}$$

There were about 578 thousand doctors in the United States in 1995.

47 b Cont There were about 578,000 total doctors.
About 554 thousand were medical and 24 thousand were osteopathy.

$$M = \frac{28,390 + 693t}{85 - t} = \frac{28,390 + 693(15)}{85 - 15} = 554 \text{ thousand}$$

$$B = \frac{776 - 12(t)}{55 - 2t} = \frac{776 - 12(15)}{55 - 2(15)} = 24 \text{ thousand}$$

48. $G_H = \frac{25}{E}$ $\quad G_C = \frac{10}{E-5}$

Where G is the gas consumed $= \frac{\text{distance}}{\text{fuel efficiency}}$

$$\text{Total } G = G_H + G_C$$

$$= \frac{25}{E} + \frac{10}{E-5}$$

$$= \frac{25(E-5)}{E(E-5)} + \frac{10E}{E(E-5)}$$

$$= \frac{25E - 125 + 10E}{E(E-5)}$$

$$\boxed{= \frac{35E - 125}{E(E-5)}}$$

49. a) Running time = $\frac{500}{c}$ Walking time = $\frac{100}{c-8}$

b) Total time = $\frac{500}{c} + \frac{100}{c-8}$

$$= \frac{500(c-8)}{c(c-8)} + \frac{100c}{c(c-8)}$$

$$= \frac{500c - 4000 + 100c}{c(c-8)}$$

$$\boxed{= \frac{600c - 4000}{c(c-8)}}$$

c) $$100 = \frac{600c - 4000}{c(c-8)}$$

$$\frac{100}{1} = \frac{600c - 4000}{c^2 - 8c}$$

$$100(c^2 - 8c) = 600c - 4000$$

$$100c^2 - 800c = 600c - 4000$$

$$100c^2 - 800c - 600c + 4000 = 0$$

$$100c^2 - 1400c + 4000 = 0$$

$$c^2 - 14c + 40 = 0$$

$$(c - 10)(c - 4) = 0$$

$c = 10$ $c = \cancel{4}$

Since you cannot burn negative calories

$\boxed{c = 10}$

Anita burns 10 calories if her total time is 100 minutes.

50. $t = \frac{K}{S}$

$195 = \frac{K}{5}$

$975 = K$

a) $t = \frac{975}{12}$

$\boxed{t = 81.25 \text{ hours}}$

It would take 12 students about 81.25 working hours to build the same kind of sailboat.

b) $75 = \frac{975}{S}$

$75S = 975$

$S = \frac{975}{75}$

$\boxed{S = 13 \text{ students}}$

It would take 13 students to build a 7 foot sailboat in 75 working hours.